A Flaw in Calculus

In this paper, I argue that there is a flaw in calculus. I begin by explaining this flaw, which is in the assumption upon which calculus rests. I then attribute this flaw to the assumption of the Cartesian Coordinate system as a frame of reference in which to analyze curved and circular areas. I then propose at least the outlines of an alternative geometry, which I call Concentric Geometry, to replace the Cartesian Coordinate system.

A Flaw in Calculus

There is a flaw in calculus. It consists in the following assumption, upon which the whole of calculus rests:

> It is possible to accurately analyze the area of curved and circular area by assuming a square or rectangular frame of reference.

The method of calculus is to analyze the area under a curve into a smaller group of rectangles. It is imagined, then, that this group of rectangles diminishes in size, ever more closely approaching the curved area. While it is true that diminishing rectangles approach the curved area, they never reach it. Hence, calculus does not succeed in accurately analyzing the area under any curve.

The terminology used to justify the claim that ever-many increasing rectangles accurately depict the area under a curve is that the rectangles approach the curve as this process goes to infinity. The problem with this terminology is that it is not possible to reach infinity. If we begin this process of diminishing rectangles today and continue it until the sun burns out in 23 billion years, we will still not reach the curve.

Calculus confuses the indefinite with the infinite. To say a process is indefinite means there is no limit to it. To say it is infinite means it is never-ending. While there is no limit to how close we can approach the area under a curve, this is a far different claim from the claim that the process is infinite. If the process is in fact an infinite one, then calculus has simply failed to capture the area under the curve. If it is an indefinite process, then calculus only gives us an approximation of the area. But giving an approximation of the area is far different from giving an accurate analysis of the area.

Another way of putting the claim of calculus is that the curve is the limit of this process as it goes to infinity. While this terminology sounds believable, it does not stand up to critical analysis. If we treat the curve as the limit of an infinite process, while admitting it can never be reached, there is an unbridgeable chasm between the end of the process and its limit. Because this chasm can never be bridged, even in principle, it is a mistake to pretend that we have given an accurate analysis of the curved area.

Even if we grant for the sake of argument that the area under a curve is the limit of an infinite process, it doesn't follow that this process accurately analyzes that area. All that follows is that we have identified an endpoint that constitutes our objective: the curve itself. Calling the curve the limit of an infinite process makes it sound as if somehow at the end of this process, this limit is reached. But because this limit cannot be reached, even in principle, treating the curve as the limit of this process adds nothing to the analysis. It simply stakes out an unreachable goal.

How could minds as great as those of Newton and Leibniz be so seriously misled? The answer is very simple. When Newton and Leibniz invented calculus, they used the Cartesian Coordinate system as their frame of reference. That is, they began with the assumption that whatever geometric figures they would analyze would be analyzed in terms of straight lines, squares, and rectangles. It is hardly surprising, then, that they had to invent the myth of an infinite process to accurately analyze the area under a curve. As anyone with common sense knows, it's not possible to fit a round peg in a square hole. Neither is it possible to accurately analyze the area under a curve by starting with straight lines, squares, and rectangles. The need for the myth of the infinite process arises only because the underlying assumption is flawed. Just as someone who lies often must tell a bigger lie to cover up his error, a mistaken assumption at the beginning requires an even more mistaken assumption at the end.

<u>An Alternative: Concentric Geometry</u>

If there is a flaw in calculus, what is the alternative? The alternative is a very simple one: abandon the Cartesian Coordinate system. While the Cartesian Coordinate system works fine for the geometry of straight lines and straight-line figures, it simply does not work for curved and circular area. In fact, curved and circular as opposed to square and rectangular areas are fundamentally incommensurable.

In place of the Cartesian Coordinate system, I propose a geometry of concentric circles. In place of straight lines, I propose placing points with specified areas on the line. These points can serve as circle origins. In fact, a point can be defined as the limiting case of a circle.

As part of this new geometry, which I propose to call Concentric Geometry, another myth must be abandoned. This is the myth that it is meaningful to speak of a point has having location only. Speaking in this way gives rise to Zeno's paradox. By supposing that it is possible to locate an object or person at a point with no area, by Zeno's paradox, no motion is possible. The way out of this paradox is to insist that units be specified up front; in the case of the paradox, that it is defined at the beginning what is to count as taking a step, or whatever the unit of motion is. Once this unit of motion is specified, the paradox disappears, since as the goal is approached, ever smaller motions will simply not reach the specified threshold. This is true, at least in theory, no matter how small the unit of motion is.

There is also a confusion embodied in the way mathematicians regard the relation between points and lines. In mathematics, a line is viewed as consisting of infinitely many points. But if a point has no area and takes up no space, then infinitely many points don't have area or take up space. Multiplying zero by infinity, if such an operation were possible, would yield zero. Again, the concept of infinity is necessary only because the initial assumption is flawed. It is necessary to compensate for the initial definition of a point as having no area by inventing the notion of infinitely many points, as if somehow that idea would compensate for the original flawed assumption.

Points lie on a line, not in the line. The failure to appreciate this distinction is partially responsible for the apparent need for the concept of infinity as applied to points. The idea that "between every two points lies another" is true only if the frame of reference has not been specified in advance. Once the frame of reference is specified, this statement is no longer true.

Zeno's paradox amounts to the idea that we can begin with one identifiable unit of measurement or frame of reference, and then continue to shift our units and frames of reference to infinity. The problem with this reasoning is that, as the unit of measurement or frame of reference is shifted, a different situation is defined. This is somewhat like shifting from Centigrade to Fahrenheit willy-nilly, or like moving from measuring in miles to measuring in rulers. Either one is fine, but it is necessary to decide ahead of time which one to use. Once the decision that a particular unit of measurement or frame of reference will be used is made, then the possibility of paradox disappears.

Concentric Geometry consists of an ever-expanding series of concentric circles. Solid, adjoining points in the form of a line replace the familiar x and y axis. These points can be as large or small as desired, but they must all be the same size. The size of these points should be determined by the needs of the situation. Points are solid and cannot overlap. In addition, the diameter of any circle as defined by these solid points will always equal the circumference divided by two.

Concentric geometry assumes as a primitive the unit of one round inch. This is precisely parallel to the assumption of the Cartesian Coordinate system of a unit of one square inch. In all likelihood, concentric geometry will be unable to provide an exact analysis of square or rectilinear area. It will, however, give exact values for circular area.

The problem of analyzing the area under a curve is not yet completely solved. It cannot yet prove that this geometry will provide exact values for curved areas. However, I believe this is possible. If we treat the area around the solid circles in this geometry as defining a primitive curve (i.e., a curve which is used as a fundamentally assumed unit), I believe it will be possible to exactly define curved area.

In this paper, I have shown that there is a flaw in calculus. This flaw is in the assumption upon which calculus rests. I then attributed this flaw to the assumption of the Cartesian Coordinate system as a frame of reference in which to analyze curved and circular areas. I then proposed at least the outlines of an alternative geometry, which I call Concentric Geometry, to replace the Cartesian Coordinate system. I believe that Concentric Geometry is a more viable frame of reference than the Cartesian Coordinate system for analyzing curved area. In addition, I believe that adopting Concentric Geometry will, in all likelihood, eliminate the need for the concept of infinity in geometry, and possibly in all mathematics.

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