Appendix A

Circular Geometry

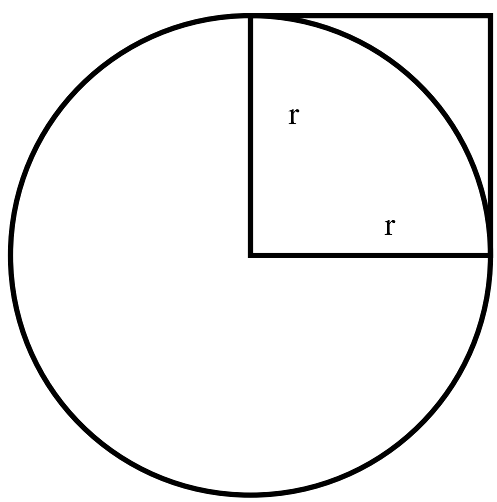
If you studied mathematics or geometry in high school or college, you probably learned the following formula for the area of circle:

$$A = \pi \times r^2$$

Here A is the area of the circle, while r is the radius of the circle. The number π, which represents the ratio of the circumference to the diameter of the circle, is an irrational number that has never been completely specified.

What is this formula actually asking us to do? If we look at the geometry of this formula, it looks like the diagram in Figure 1.

Figure A-1. The Square Inch as the Unit of Measure for Circular Area



The value r^2 gives the geometric area of the square in the above diagram. The formula for the area of a circle, then tells us that π squares with sides equal to radius r fit into the area of a circle with radius r.

What, if anything, is the problem with this formula? And why do we need to have π in the formula? The reason for π is that there is no definite number of times that a square can fit inside a circle. It is often said “You can’t fit a square peg into a round hole.” This common saying reflects the insight that the area of a square cannot be used as a unit of measurement for circular area. Since there is no definite number of times that a square will fit inside a circle, the value π has to be included to create a usable formula for circular area.

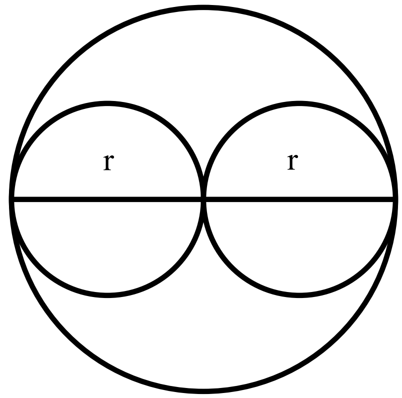
The relation between circular area and square area is that they are incommensurable. What this means is that they cannot be both measured exactly using the same standard or unit of measure. Straight lines and squares work fine for squares and rectangles, but they do not allow us to provide exact values for the areas of circles.

If the areas of squares and circles cannot be measured exactly using the same unit of measurement, we have several choices. One is to continue as we are, using square area as the unit of measurement for circular area. This has the advantage of familiarity, provided we don’t mind using π. A second alternative is to use a different unit of measurement for circular area. This is the alternative I would like to suggest here.

An Alternative Unit of Measure

As an alternative unit of measure for circular area, I suggest the round inch. A round inch is a circle with a diameter of one inch. If we use the round inch as the unit of measure for circular area, this unit of measure looks like the drawing in Figure 2.

Figure A-2. The Round Inch is the Unit of Measurement for Circular Area



In Figure 2, each of the two smaller circles is equal to A/4, where A = the area of the circle. Each small circle has a diameter of one inch, and so is equal to one round inch. The area of the large circle is equal to $(\text{diameter})^2$. This value is equal to 2^2, which is equal to 4. So the circle above has an area of four round inches.

Circular Mils

A similar approach to this already exists for measuring the area of round wire. In order to avoid using decimals, the area of round wire is often measured in circular mils. The area of a circular mil is $(\text{diameter})^2$, so the formula is as follows:

$$A = (\text{diameter})^2$$

A mil is equal to $1/1000^{th}$ of an inch (0.001 inch). A circle that has as one mil as its diameter has an area of $1^2 = 1$. A circle that has a diameter of 4 mils has an area of $4^2 = 16$ circular mils.

What is the relation between a round inch and circular mils? A round inch has a diameter of one inch. Since a mil is $1/1000^{th}$ of an inch, a round inch has a diameter of 1000 mils. This means that a round inch has an area of 1000^2 circular mils, or 1,000,000 circular mils. So any area that can be measured in round inches can be measured in circular mils, and vice versa.

Application

What is the application for circular geometry? Circular geometry can be used anywhere someone wants to measure circular area. It is true that many buildings and other structures are either square or rectangular. This is an example of geometry influencing architecture. Because most of the geometry that is taught in schools is some uncompromisingly linear, meaning that it is based on straight lines, squares, and rectangles, many of the buildings and other structures that are created using this geometry reflect its underlying linear nature. On the other hand, if we look in nature, we find a wide assortment of waves, curves, circles, and other nonlinear geometric shapes. The world is round, even though it looks flat, and many natural shapes are nonlinear as well.

One area that circular geometry has an application is in flow measurement. Pipes are round, and it is often necessary to determine the area of a pipe in order to determine volumetric flow. The formula that is usually used is the following one:

$$Q = A \times v$$

In the above formula, Q is equal to volumetric flow, A is the cross-sectional area of the pipe, and v is the average velocity of the fluid. It is in providing the cross-sectional area of a pipe that the value of π is used in calculating flowrate. If this area is provided in round inches rather than square inches, flowrate can be calculated without the use of π.

Round inches can generally be substituted for square inches in geometry when calculating the area of circles. Just as circular mils are used to measure the area of wire, so round inches can be used to measure the areas of circles. Of course, just as there is no exact way to measure circular areas in terms of straight lines, there is no way to exactly measure the area of squares and rectangles using circular geometry. Just as a hammer is used for nails, and a screwdriver is used for screws, so each type of geometric structure requires its own geometry.